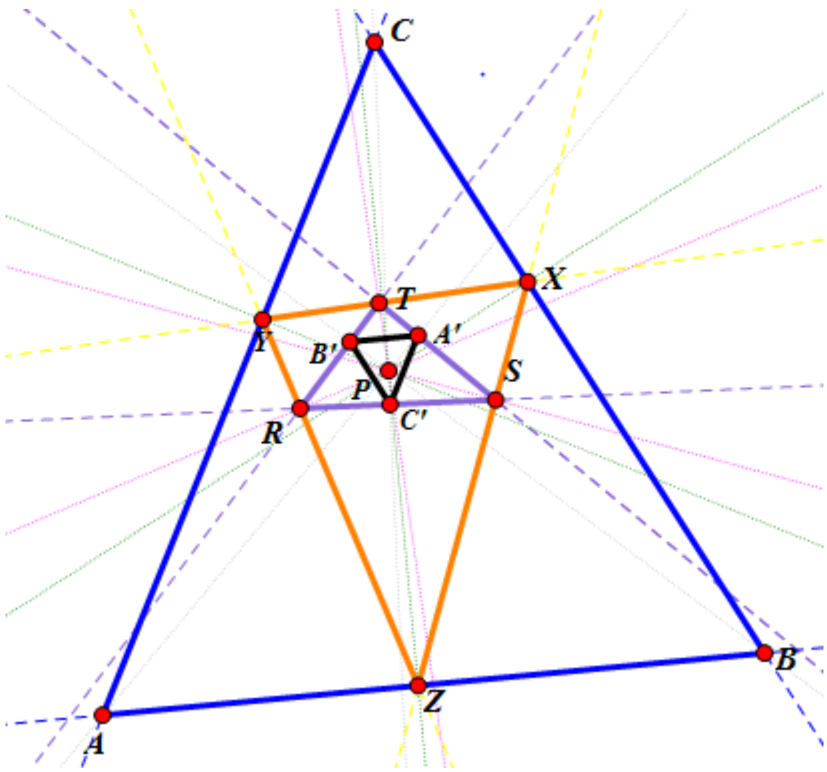
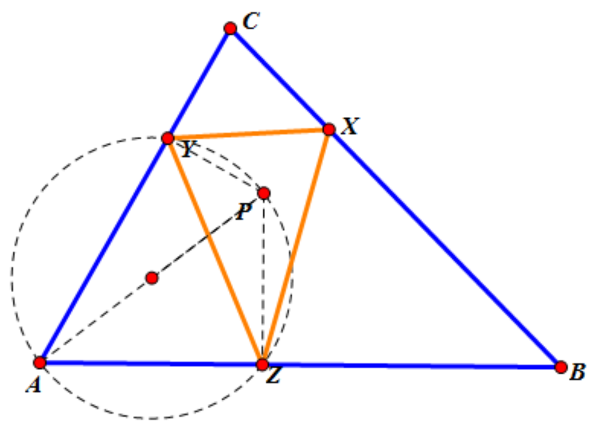
The University of Georgia

Mathematics Education
EMAT 4680/6680 Mathematics with Technology
Jim Wilson, Instructor

Prove the pedal triangle of the pedal triangle of the pedal triangle of a point is similar to the original triangle. That is, show that the pedal triangle of A'B'C' of pedal triangle RST of pedal triangle XYZ of pedal point P is similar to triangle ABC.

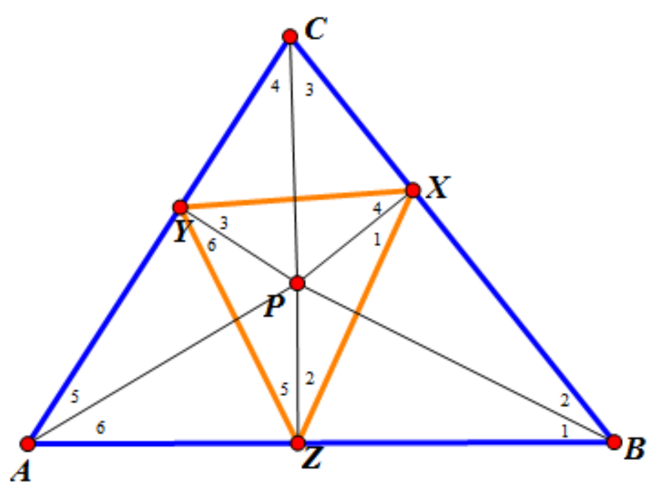




Let us first take a look at the pedal triangle XYZ with pedal point p of triangle ABC. We know that <PYA and <PZA are right angles because of the nature in which the pedal triangle is formed, i.e. perpendicular to our original triangle ABC. Take a look at the circle drawn in the figure above. AP is the diameter of the circle. The rays of <PYZ intersect the endpoints of the arc or diameter at A and P. The rays of <PZA also intersect the endpoints of the arc or diameter at A and P. We also know that arcs of equal length subtend equal angles. Therefore the length of PY is equal to the length of PZ, and the length of YA is equal to the length of ZA.

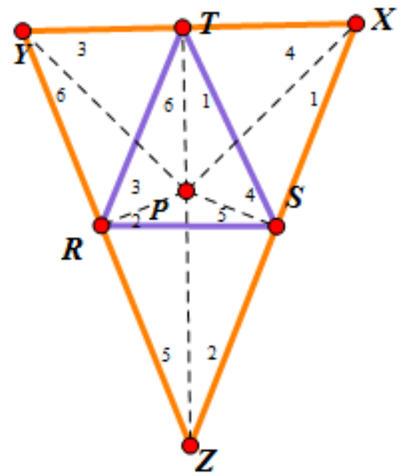
We can also use similar logic to make other conjectures about this diagram. Both the rays of <PZY and <PAY intersect the endpoints of chord PY. In addition, both the rays of<PAZ and <PYZ intersect the endpoints of chord PZ.

Here is a diagram with numbers representing congruent angles.

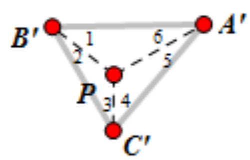


We continue using the same logic as was applied with the first pedal triangle.

For the second pedal triangle:



For the third pedal triangle:



We have already shown that <PAC=<PZY. By similar logic <PAC=<PZY=<PSR=<PA'C'
We have already shown that <PAB=<PYZ. By similar logic <PYZ=<PTR=<PA'B'.
We can also check to see this by measuring the respective angles in GSP.
Therefore A'B'C' is similar to ABC because they have corresponding congruent angles.
In addition we can see this from our diagrams with numbers representing the angles.
<A'B'C'=1+2=<ABC
<B'C'A'=3+4=<BCA
<C'A'B'=5+6=<CAB
So, A'B'C' is similar to ABC.